ALGEBRAIC REASONING – Year 1

PARTITIONING
Red Apples, Green Apples

Abstract
This task asks students to explore the different ways that numbers can be partitioned into two parts. Working systematically, students are asked to find and show that they have all possibilities. This task affords an exploration of early algebraic thinking and scaffolds the students towards seeing a pattern and making a generalisation regarding all possible combinations.

Mathematical Purpose
This task explores the concept of equivalence. The equals sign is presented as a symbol that indicates balanced value on both sides of an equation. Students explore equivalence through the partitioning of numbers into two parts. Opportunity to explore the commutative property and compensation also develops through the task. Students work systematically to find all possible combinations.

Australian Curriculum: Mathematics Year 1
Number & Algebra Count collections to 100 by partitioning numbers using place value (ACMNA014)
Patterns and Algebra Describe patterns with numbers and identify missing elements (ACMNA035)

This is a stand-alone task.

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TASK

Pose the question:

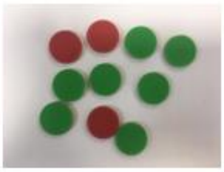
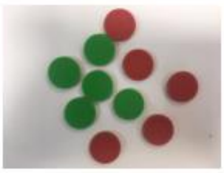
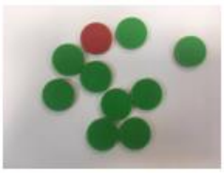
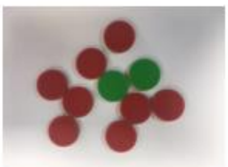
A shop was selling red and green apples. I bought ten apples.
How many red apples and how many green apples might I have bought?

EXPLORE

Provide students with 10 two colour counters. Explain to the students that the counters represent the apples. The counters will help work out the different combinations of apples that might be in the bag. It is important that students associate two different colours of apples with the colours that are on the counters. If red and green counters are not available, make sure students are happy that one of the colours represents the red apples and the other colour represents the green apples.

If two colour counters are not available, a red and a green sticky dot could be placed on either side of a counter. Students could also draw 10 cubes/counters out of a bag that contains at least 10 red and 10 green cubes/counters.

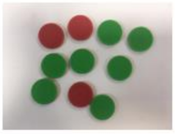
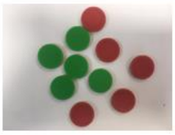
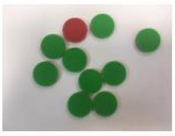
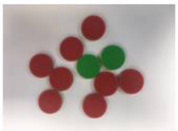
Students toss the 10 counters and record how many of each colour there are. Have students record their results.

	Red Apples	Green Apples
	6	4
	3	7
	5	5
	1	9
	8	2

Enabling Prompt: Explore the different combinations for 5 or 6 apples in a bag.

Extending Prompt: Can you work out some different combinations without tossing the counters?

Ask students to share some of the results that they have collected. These results can be recorded on the board or a class display. At this point, record the results randomly. Look at how these can be recorded as an equation.

	Red Apples	Green Apples	
	6	4	→ 6 + 4 = 10
	3	7	→ 3 + 7 = 10
	5	5	→ 5 + 5 = 10
	1	9	→ 1 + 9 = 10
	8	2	→ 8 + 2 = 10

This introduces the important concept of ***equivalence***. All these combinations have the value as each other.

Pose the questions:

How many different combinations are possible?
How do we know if we have found them all?

Ask students to explore whether they have found all possible combinations. Students should record results as an equation.

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Questioning to direct the investigation and challenge students thinking and reasoning:

What is similar and what is different about the combinations that you have found?

- Students will see that some combinations look similar, e.g. 4 red apples and 6 green apples is similar to 6 red apples and 4 green apples. This is an example of the commutative property. It is important that students recognise that these are different. Each of the combinations has a commutative combination.
- The different combinations are also similar in that they all add to 10. While this may seem obvious, it is important that students realise that, while the numbers are different, the value of the equation is the same. This is exploring the concept of equivalence and the meaning of the equals sign.

How can you order the combinations that you have collected to help you see if you have found them all?

- Ordering the combinations will help show which are missing.

Extending prompt: *What if I bought a bag of red apples, green apples and pears. In the bag are 10 pieces of fruit. Are these combinations equivalent? Are they still equivalent to the bags with the bags of apples?*

- Students can explore equivalence between combinations of 2 or 3 numbers that add to 10. For example,

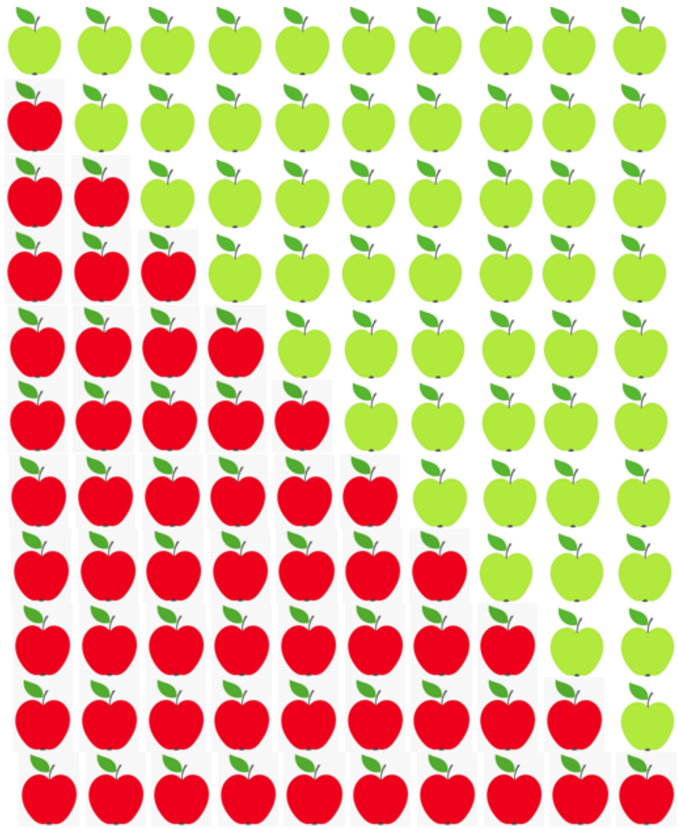
$$2 + 3 + 5 = 1 + 6 + 3$$
$$4 + 6 = 2 + 3 + 5$$

REFLECT

Select some students to present their work to the class. Look at the way the combinations have been ordered.

Students can also organise the combinations in ascending or descending order. Ordering in this manner easily identifies any missing combinations.

The following is ordered according to the ascending order of red apples:



0 + 10
1 + 9
2 + 8
3 + 7
4 + 6
5 + 5
6 + 4
7 + 3
8 + 2
9 + 1
10 + 0

Recording in this way also highlights the concept of ***compensation***. A new combination is made by taking a green apple and replacing it with a red apple creates a new combination.

Explore the commutative property of addition illustrated in the task:

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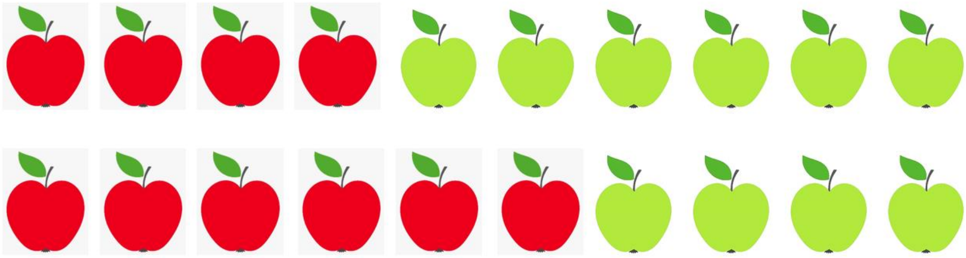
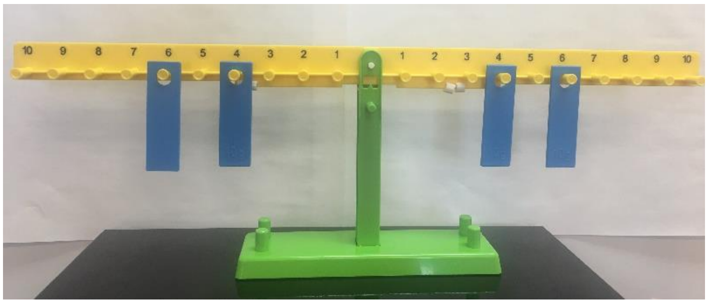
Although the same numbers have been used, the combinations of apples are different.



$6 + 4 = 4 + 6$

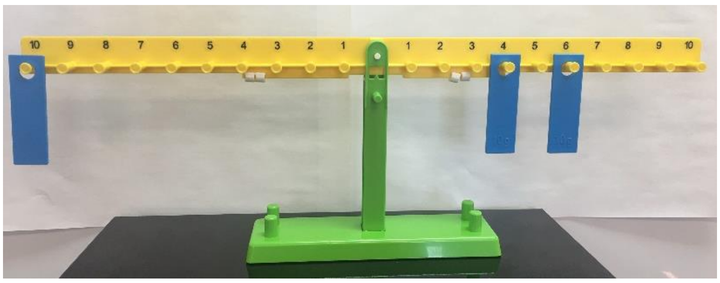
This can also be shown on the number balance:



$6 + 4 = 4 + 6$

Look at how the equations recorded are equivalent, that is, has the same value as each other.

Equivalence can be represented as equations and using a number balance:



$10 = 6 + 4$

Note: we know there are 10 apples in total and so the 10 is recorded first. This is important that students are confident that this equation is true and helps to dispel the myth that equals means ‘give the answer’. It highlights that the equals sign represents balance, that is, both sides of the equation have the same value.

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6 + 4 = 1 + 9

6 + 4 = 3 + 7

Pose the question:

What is there were 6 apples in the bag? 12 apples? 17 apples?
How could we find the combinations for any number of apples in the bag?

This encourages students to move towards a generalisation. All possible two number partitions for any whole number is always one more than the number itself. So, for 10, 11 different two number partitions can be made. These partitions can be found by listing the combinations systematically.

Students can explore the possible partitions for different numbers. Have students explain how all the two number partitions can be found.